Factor Income Taxation, Growth, and Investment Specific Technological Change*

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February 27, 2014

Abstract

We construct a tractable endogenous growth model with endogenous investment specific technological change (ISTC) to explain why advanced economies with similar growth rates have widely varying factor income tax rates. Public and private capital stock externalities are assumed to augment ISTC. A specialized labor input augments final good production. We show that several labor and capital tax combinations can implement the planner's growth rate on the balanced growth path. We show that allowing for endogenous ISTC and externalities leads to a divergence between the welfare maximizing factor income tax mix and the factor income tax combination that implements the planner's allocations. A simple numerical exercise offers an explanation for how the trade-off between factor income taxes is affected by the magnitude of the externalities.

Keywords : Investment Specific Technological Change, Factor Income Taxation, Endogenous Growth, Fiscal Policy.

JEL Codes: E2; E6; H2; O4

*We thank Ken Kletzer, Partha Sen, Aditya Goenka, Javed I. Ahmed, Joydeep Bhattacharya, Chetan Dave, Alok Johri, Premachandra Athukorala, Rajesh Singh, Prabal Roy Chowdhury, Noritaka Maebayashi and seminar participants at the 7th Annual Growth and Development Conference (ISI -Delhi), ISI Kolkata, the Australian National University, the 11th Louis-André Gérard-Varet Conference (Marseilles), and the December 2012 Asian Meeting of the Econometric Society (New Delhi) for insightful comments.

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1 Introduction

Why do advanced economies with roughly identical growth rates have widely varying factor income tax rates? In this paper, we develop a growth model with endogenous investment specific technological change and production externalities to understand this question.

Figure (1) plots the average annual real GDP growth rate from 1990 to 2007 against the factor income tax ratio for several advanced economies.[1] Average growth for all countries (excluding Ireland) falls between 0.875% and 2.462%. The standard deviation of the average real GDP growth rates is 0.878 (excluding Ireland, the standard deviation is 0.4756) which indicates low dispersion of growth rates. What is striking however is that the range in the ratios of the average capital income tax rate to the average labor income tax rate in these economies is much more pronounced: 0.3951 to 1.725.[2] In other words, there is more dispersion in factor income tax ratios relative to dispersion in growth. Also, for 12 out of 17 economies the tax on labor income is higher than the tax on capital income

Figure (2) plots the difference between the average factor income tax rates for these economies. Despite having similar growth rates, what is striking is that whereas the difference between factor income taxes is large in some countries, it is quite small in others.[3]

[Insert Figure 1 and 2]

Finally, Figure (3) plots the levels of factor income tax rates across the G7 countries. The incidence of factor income taxation is quite disparate. In the US, UK, Canada, and Japan, the tax on capital income is greater than the tax on labor income. In contrast, for Germany, Italy, and France, the reverse is true.

[Insert Figure 3]

[1] The growth rates are calculated from the OECD (2012) database: see Table ($VXVOB$). The countries are: Austria (AUS), Belgium (BEL), Canada (CAN), Denmark (DEN), Finland (FIN), France (FRA), Germany (GER), Greece (GRE), Ireland (IRE), Italy (ITA), Japan (JPN), Netherlands (NET), Portugal (PRT), Spain (SP), Sweden (SWE), United Kingdom (UK) and United States of America (USA). The base year is 2000

[2] Canada and Japan have data on capital and labor income tax estimates based on the approach used in Mendoza et al. (1994) and Trabandt and Uhlig (2009) from 1965 to 1996. For Germany, United Kingdom and United States of America, data is from 1965 to 2007. For France, the data is from 1970 to 2007. For Italy, the data is from 1980 to 2007. For Austria, Belgium, Denmark, Finland, Netherlands, Portugal and Sweden, the data is from from 1995 to 2007. For Spain and Greece, the data is from 2000 to 2007. Finally, for Ireland, the data is from 2002 to 2007.

[3] The data on factor income taxes are from Mendoza et al. (1994) and Trabandt and Uhlig (2009). The latter have used the approach in Mendoza et al. (1994) to estimate the tax rates for 17 OECD nations till 2007.

To explain these observations, we construct an endogenous growth model with endogenous investment specific technological change.[4] The point of departure of our model however is that public capital – financed by distortionary taxes – augments investment specific technological change (ISTC) *as a positive externality.*[5] Typically in the literature, the public input is seen as directly affecting final production directly either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). We therefore formalize the link between factor income taxation and growth through the effect that public policy has on investment specific technological change.[6]

In addition to positive spillovers from the public capital stock, we assume two other externalities. First, we assume that private capital externalities also affects investment specific technological change. This assumption is motivated by Greenwood et al. (1997), who show that the real price of capital equipment in the US – since 1950 – has fallen alongside a rise in the investment-GNP ratio; hence, we assume that the aggregate stock of capital also exhibits a positive externality in investment specific technological change through the aggregate capital output ratio. Greenwood et al. (1997, p. 342) say: "The negative co-movement between price and quantity.....can be interpreted as evidence that there has been significant technological change in the production of new equipment. Technological advances have made equipment less expensive, triggering increases in the accumulation of equipment both in the short and long run."[7] Second, we assume that the specialized labor input in the

[4] A growing literature has attributed the importance of investment specific technological change to long run growth (see Greenwood et al. (1997, 2000); Whelan (2003)). Investment specific technological change refers to technological change which reduces the real price of capital goods. Greenwood et al. (1997, 2000) show that once the falling price of real capital goods is taken into account, this explains most of the observed growth in output in the US, with relatively little being left over to be explained by total factor productivity. Other authors, such as Gort et al. (1999) distinguish between equipment specific technological change and structure specific technological change. These authors show that 15% of US economic growth rate can be attributed to structure specific technological change in the post war period, while equipment-specific technological progress accounts for 37% of US growth. This implies 52% of US economic growth can be attributed to technological progress in new capital goods.

[5] Our setup *also* allows investment specific technological change to enhance the accumulation of public capital. For instance, providing better infrastructure today reduces the cost of providing public capital in the future.

[6] To the best of our knowledge, we are not aware of any paper in the literature in which public capital affects ISTC, either directly or as an externality. In a different context, Harrison and Weder (2000) build a two sector representative agent model with increasing returns to scale driven by externalities that come from sector specific as well as aggregate economic activity. Benhabib and Farmer (1996) show that small empirically plausible external effects lead to indeterminacy. Neither of these papers has a role for public capital. Lloyd-Braga, Modesto, and Seegumuller (2008) introduce positive government spending externalities in preferences. In our model, externalities from the public stock influence ISTC directly.

[7] In addition, DeLong and Summers (1991) show that investment in machinery is associated with very strong positive externalities. Hamilton and Monteagudo (1998) find that capital is associated with positive external effects in an estimated Solow growth model.

research sector exerts a positive externality in the production of the first sector, the final good.[8] We show that a higher weight on the specialized labor input externality raises the growth rate on the balanced growth path and gives us additional traction in explaining the factor income tax gaps documented in Figures (1), (2) and (3).

1.1 Description of the Model and Main Results

In our model, the final good sector produces a final good, using private capital, and labor. Labor supply is composite in the sense that one type of labor activity is devoted to final good production, and the other to research which directly reduces the real price of capital goods in the next period. The agent optimally chooses each labor activity. The second sector captures the effect of public capital and the private capital stock spillovers and research activity on reducing the real price of capital goods. The planner is assumed to internalize the externalities. In the planner's problem, we assume that public investment is financed by a *fixed* proportional income tax as in Barro (1990). Because the tax rate is fixed, this characterizes the constrained first best fiscal policy in our model. For fixed parameters, we show that there is an optimal growth rate that results from solving the planner's problem. We characterize the steady state balanced growth path for this economy. We show that the growth maximizing tax rate is determined by the relative importance of the public capital output ratio vis-a-vis the private capital output ratio in the investment specific technological change function. The implication of this is that if a planner was to choose the tax rate to maximize balanced growth, the planner could maximize long run growth as long as the tax rate equals the relative contribution of public capital to investment specific technological change.[9]

We then decentralize the planner's allocations. We assume that public investment is financed by distortionary factor income taxes on capital and labor income. We show that while the constrained first best fiscal policy can be implemented as a competitive equilibrium, there is an indeterminate combination of capital tax rates and the labor tax rates that can replicate the planner's allocations. Our definition of indeterminacy is as follows: there is *no unique combination* of factor income taxes on capital *and* labor income that imple-

[8] A real life example that motivates this assumption is the skill required for advanced manufacturing jobs. Skilled factory workers today are typically "hybrid-workers": they are both machinists as well as computer programmers. For instance, in the US metal-fabricating sector, workers not only use cutting tools to shape a raw piece of metal, but they also write the computer code that instructs the machine to increase the speed of such operations. See Davidson (2012).

[9] In a full blown planner's problem, the planner would be allowed to control the accumulation of public capital. In this case, we will show later that the optimal tax rate is also constant. By focusing on the constrained first best, we are able to keep the model tractable without any loss of generality.

ments the planner's allocations. Indeterminacy obtains because the planner's allocations are implemented with a fixed tax rate on output.

The main insights that we gain from the model are as follows:

- When there are no externalities, *equal* factor income taxes always yield the optimal growth rate from the planner's problem.

- In the presence of externalities, there are various factor income tax rates that implement the planner's growth rate along the steady state balanced growth path. Intuitively, the *higher* is the externality associated with the specialized labor input in the research sector (which exerts an externality in the production of the first sector, the final good), the *lower* is the optimal tax on capital for a *given* tax on labor income. This is because agents - by taking this externality as given - under-fund capital accumulation. A lower tax on capital income incentivizes capital accumulation and restores the planner's growth rate. The difference between both factor income taxes declines as the effect of the externality is reduced. Similarly, when the externality effects from the aggregate stocks (public and private) increase, these stocks increase the level of investment specific technological change. However, since agents do not internalize these spillovers from the aggregate stocks, they under-fund capital accumulation relative to the efficient growth rate. To incentivize capital accumulation, the planner sets a low optimal tax on capital income. Our calibrated results show that the trade-off between factor income taxes is affected by the magnitude of the externalities.

- We show that the divergence of the welfare maximizing factor income tax mix from the factor income tax mix that implements the planner's allocations can be decomposed into two effects: 1) the effect because of externalities from public and private capital, and 2) the effect on final good production due to positive spillovers from specialized labor. In the limiting case, where the externalities go to zero, and when ISTC is exogenous, the welfare maximizing factor income tax mix converges to the growth maximizing factor income tax mix. Hence, both production externalities and allowing for endogenous ISTC imply departures from the planner's allocations.

1.2 Literature Review

The setup of our model is technically similar to Huffman (2007, 2008) who explicitly models the mechanism by which the real price of capital falls when investment specific technological change occurs. Huffman (2008) builds a neoclassical growth model with investment specific technological change. Labor is used in research activities in order to increase investment specific technological change. In particular, the changing relative price of capital is driven by research activity, undertaken by labor effort. Higher research spending in one period lowers the cost of producing the capital good in the next period.[10] Investment specific technological change is thus endogenous in the model, since employment can either be undertaken in a research sector or a production sector. His model includes capital taxes, labor taxes, and investment subsidies that are used to finance a lump-sum transfer. Huffman (2008) finds that a positive capital tax that is larger than a positive investment subsidy along with zero labor tax can replicate the first best allocation. Huffman's models however do not incorporate public capital - a feature we show that is important in matching factor income tax rates observed in advanced economies.

Our paper is also related to two other strands of the literature on fiscal policy and long run growth in the neoclassical framework. The first literature - started by Barro (1990) and Futagami, Morita, and Shibata (1993) – incorporate a public input – such as public infrastructure – that directly augments production. In Barro (1990), public services are a flow; while in Futagami, Morita, and Shibata (1993), public capital accumulates. However, in the large literature on public capital and its impact on growth spawned by these papers, the public input, whether it is modeled as a flow or a stock, doesn't directly influence the real price of capital goods.[11] Since public capital affects the real price of capital explicitly in our model, this means that the public input affects future output through its effect on both future investment specific technological change, as well as future private capital accumulation. Our main methodological contribution is that we merge the public capital/endogenous growth literature with the endogenous investment specific technological change literature. To the best of knowledge, whereas distortionary taxes have been exogenously imposed to correct for externalities in the literature, our model is the first attempt to explain how differences in factor income taxes across countries can be explained by the *existence* of production externalities. To the extent that such spillovers exist in actual economies, our analysis seeks to understand how the magnitude of various externalities/spillovers have a bearing on the optimal factor income tax mix that implements the constrained first best optimum on a balanced growth path.

The rest of the paper proceeds as follows. Section 2 develops the basic model structure followed by characterizing the planner's model, the competitive equilibrium and some numerical experiment under unequal factor income taxes that shows how the magnitude of

[10] Krusell (1998) also builds a model in which the decline in the relative price of equipment capital is a result of R&D decisions at the level of private firms.

[11] For instance, in Ott and Turnovsky (2006) - who use the flow of public services to model the public input - and Chen (2006), Fischer and Turnovsky (1998) - who use stock of public capital - the shadow price of private capital is a function of public and private capital.

externalities in the model is crucial to the optimal tax mix. Section 3 concludes.

2 The Model

Consider an economy that is populated by identical representative agents, who at each period t, derive utility from consumption of the final good C_t and leisure $(1 - n_t)$. The term n_t represents the fraction of time spent at time t in employment. The discounted life-time utility, U, of an infinitely lived representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_t)]. \tag{1}$$

where $\beta \in (0, 1)$ denotes the period-wise discount factor. There is no population growth in the economy and the total supply of labor for the representative agent at any time t is given by n_t such that

$$n_t \equiv n_{1t} + n_{2t}, \tag{2}$$

where n_{1t} is labor allocated for final goods production and n_{2t} is labor allocated for enhancing investment specific technological change. The representative agent however is not aware that his allocation of labor towards n_{2t} also influences productivity of final goods production.

The final good is therefore produced by a standard production function with capital K_t, n_{1t}, and aggregate n_{2t} entering as an externality, which we denote by $\overline{n}_{2t}$. The key difference is that the planner internalizes the externality from n_2 in direct production, while agents do not. The production function is given by

$$Y_t = \underline{A} K_t^{\alpha} n_{1t}^{1-\alpha} \underbrace{\left(n_{2t}^{1-\alpha}\right)^{\xi}}_{\text{Externality}} \tag{3}$$

where $\underline{A} > 0$ is a scalar that denotes the exogenous level of productivity, $\alpha \in (0, 1)$ is the share of output paid to capital, and $\xi > 0$ is the externality parameter capturing the effect that n_2 has on direct production. When $\xi > 0$, the planner internalizes the effect that n_2 has on direct production. When $\xi = 0$, there is no externality from n_2 on the production of the final good. Note, in this framework, as in Huffman (2008) the two labor activities n_{1t} and n_{2t} are assumed to be equally skilled, but are optimally allocated across different activities by households.[12]

[12] Other papers in the literature - such as Reis (2011) - also assume two types of labor affecting production. In Reis (2011), one form of labor is the standard labor input, while the other labor input is entrepreneurial labor.

Private capital accumulation grows according to the standard law of motion augmented by investment specific technological change,

$$K_{t+1} = (1-\delta)K_t + I_t Z_t, \tag{4}$$

where $\delta \in [0,1]$ denotes the rate of depreciation of capital and I_t represents the amount of total output allocated towards private investment at time period t. We assume that, $\delta = 1$, to keep the model tractable. Z_t represents investment-specific technological change. The higher the value of Z_t, the lower is the cost of accumulating capital in the future. Hence Z_t can also be viewed as the inverse of the price of per-unit private capital at time period t. The term, $I_t Z_t$, therefore represents the effective amount of investment driving capital accumulation in time period $t+1$.

In addition to labor time deployed by the representative firm towards R&D, the public capital stock, G, plays a crucial role in lowering the price of capital accumulation. Typically the public input is seen as directly affecting final production – either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). Instead, here we assume that the public input facilitates investment specific technological change. This means that the public input affects future output through future private capital accumulation directly. In the above literature, the public input affects current output directly. This is our point of departure. We therefore formalize the link between factor income taxation and growth through the effect that public policy has on investment specific technological change.

We assume that in every period, public investment is funded by total tax revenue. Public capital therefore evolves according to

$$G_{t+1} = (1-\delta)G_t + I_t^g Z_t, \tag{5}$$

where G_{t+1} denotes the public capital stock in $t+1$, and I_t^g denotes the level of public investment made by the government in time period t:

$$I_t^g = \tau Y_t, \tag{6}$$

where $\tau \in (0,1)$ is the proportional tax rate.[13] We assume that Z_t augments I_t^g in the same way as I_t since it enables us to analyze the joint endogeneity of Z and G. To derive the balanced growth path, we further assume that the period wise depreciation rate $\delta \in [0,1]$ is

[13] Since $\delta = 1$, equation (5) implies that $G_{t+1} = I_t^g Z_t$, i.e., the ISTC adjusted public investment (flow) at period t equals the public capital stock in $t+1$.

same for both private capital and public capital.

2.1 Investment Specific Technological Change

To capture the effect of public capital on research and development, we assume that Z grows according to the following law of motion,

$$Z_{t+1} = B{n_{2t}}^{\theta} Z_t^{\gamma} \underbrace{\left\{ \left(\frac{G_t}{Y_{t-1}} \right)^{\mu} \left(\frac{K_t}{Y_{t-1}} \right)^{1-\mu} \right\}^{1-\gamma}}_{\text{Externality}}. \tag{7}$$

Here, B stands for an exogenously fixed scale productivity parameter and $\mu \in (0, 1)$ captures the impact of public investments on investment specific technological change. We assume that the parameters, $\theta \in (0, 1)$ and $\gamma \in (0, 1)$; θ stands for the weight attached to research effort and γ is the level of persistence the current year's level of technology has on reducing the price of capital accumulation in the future.[14] The term $\frac{G_t}{Y_{t-1}}$ represents the externality from public capital in enhancing investment specific technological change in time period $t+1$. The aggregate capital-output ratio, $\frac{K_t}{Y_{t-1}}$, is also assumed to exert a positive externality effect on investment specific technological change. In particular, a higher aggregate stock of capital in t, K_t, relative to Y_{t-1}, raises Z_{t+1}. Like the externality from n_2, the planner internalizes the effect that stock of public capital and private capital has on investment specific technological change, while agents treat the effect of $\frac{G_t}{Y_{t-1}}$ and $\frac{K_t}{Y_{t-1}}$ on Z_{t+1} – the bracketed term – as given.[15] Note that when $\gamma = 1, \theta = 0$, ISTC is exogenous.

[14] This contrasts with Huffman (2008) where $\gamma = 1$ is required for growth rates of Z and output to be along the balanced growth path. We require $\gamma \in (0, 1)$ for the equilibrium growth rate to adjust to the steady state balanced growth path.

[15] We assume that $\delta = 1$ for analytical tractability. Our assumption of $\frac{G_t}{Y_{t-1}}$ augmenting Z_{t+1} is for two reasons. First, if G_t augmented output Y_t instead, we can show that in equilibrium, the only possible balanced growth path is when the gross growth rate of all endogenous variables is 1 that is, all variables are at their steady state. This means, public capital will not affect the growth rate. Hence, allowing for ISTC to depend on the public input enables the balanced growth path to be affected by tax policy *through* ISTC. Second, if Z_{t+1} was instead parametrized as

$$Z_{t+1} = B{n_{2t}}^{\theta} Z_t^{\gamma} \left\{ G_t^{\mu} K_t^{1-\mu} \right\}^{1-\gamma},$$

i.e., G and K are not normalized by Y, we can show that the growth rate if Z will never converge to a balanced growth path.

2.2 The Planner's Problem

We first solve the planner's problem. The aggregate resource constraint the economy faces in each time period t is given by

$$C_t + I_t \equiv Y_t(1-\tau) = \underline{A}K_t^{\alpha} n_{1t}^{1-\alpha}\left(n_{2t}^{1-\alpha}\right)^{\xi}(1-\tau) \tag{8}$$

where agents consume C_t at time period t and invest I_t at time period t. Aggregate consumption and investment add up to after-tax levels of output, $Y_t(1-\tau)$, where $\tau \in [0,1]$ is the proportional tax rate that is assumed to be fixed in every time period.

The planner maximizes life-time utility of a representative agent – given by (1) – subject to the economy wide resource constraint given by (8), the laws of motion (4) and (5), the equation describing investment specific technological change (7), the identity for total supply of labor given by (2) and finally, the government budget constraint given by (6).[16] Because the tax rate is fixed, this yields the constrained first best fiscal policy in the model.[17]

2.2.1 First Order Conditions

The Lagrangian for the planner's problem is given by,

$$L = \sum_{t=0}^{\infty}\beta^t[\log C_t + \log(1-n_{1t}-n_{2t}) + \lambda_t\{\underline{A}K_t^{\alpha}n_{1t}^{1-\alpha}\left(n_{2t}^{1-\alpha}\right)^{\xi}(1-\tau) - C_t - I_t\}]. \tag{9}$$

where λ_t is the Lagrangian multiplier. Because our focus is on the balanced growth path, we assume that $\delta = 1$.

The following first order conditions obtain with respect to C_t, K_{t+1}, n_{1t}, and n_{2t}, respectively[18]:

$$\{C_t\} : \tfrac{1}{C_t} = \lambda_t \tag{10}$$

$$\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}} + \underbrace{\beta^2\frac{I_{t+2}}{C_{t+2}}\frac{(1-\gamma)(1-\mu)}{K_{t+1}} + \frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha)}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{C_{t+j+3}}}_{\text{Additional term due to externality in ISTC}} \tag{11}$$

[16] Clearly, $C_t + I_t + I_t^g = Y_t$.

[17] As mentioned in the introduction, by focusing on the constrained first best, we are able to focus our numerical results on the contribution of public capital to ISTC. In a full blown planner's problem, the planner would instead be allowed to control the accumulation of public capital. While we do not show this here, this would yield that the growth rate is maximized at $\tau_t = \varkappa\mu$, for all t where we can show that $\varkappa < 1$. Hence, the first best also yields a constant tax rate. These results are available from the authors on request.

[18] See Appendix A for derivations.

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau)}{C_t n_{1t}} - \underbrace{\frac{\beta^2(1-\alpha)(1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{C_{t+j+2}}}_{\text{Additional term due to externality in ISTC}} \tag{12}$$

and,

$$\{n_{2t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)\xi Y_t(1-\tau)}{C_t n_{2t}} + \underbrace{\beta\theta\frac{I_{t+1}}{C_{t+1}n_{2t}} + \frac{\beta^2(\gamma\theta-(1-\alpha)\xi(1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{C_{t+j+2}}}_{\text{Additional term due to the joint effect of endog. ISTC and externalities}}. \tag{13}$$

Equation (10) represents the standard first order condition for consumption, equating the marginal utility of consumption to the shadow price of wealth. Equation (11) is an augmented form of the standard Euler equation governing the consumption-savings decision of the household. The first term on the RHS of equation (11), $\frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}}$, corresponds to the after tax marginal productivity of capital in $t+1$. The second term, $\frac{\beta^2 I_{t+2}(1-\gamma)(1-\mu)}{C_{t+2}K_{t+1}} > 0$, is the (further) increment to the marginal productivity of capital that agents get in period $t+2$ by postponing consumption today. This is increasing in the investment-consumption ratio, but adjusted by the weight, $1-\mu$, of the aggregate capital-output ratio, in the investment specific technological change equation. The third term, $\frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha)}{K_{t+1}} \sum_{j=0}^{\infty} \frac{\beta^j\gamma^j I_{t+j+3}}{C_{t+j+3}}$, is the discounted increase in marginal productivity of investing in capital from period $t+3$ onwards. This expression is adjusted by the term $(\gamma(1-\mu)-\alpha)$, which can be either positive or negative – depending on the relative importance of capital in equation (7) vis-a-vis its direct contribution to increasing output, from (3). It is easy to see that when $\gamma = 1$, the additional terms in the Euler equation are equal to zero, yielding the standard Euler equation.

Equation (12) denotes the optimization condition with respect to labor supply (n_{1t}). Since $0 < \gamma < 1$, the second term in the RHS is positive which constitutes a reduction in the marginal utility of leisure. This reduces n_1 relative to the standard case in which there is no investment specific technological change. Similarly, in equation (13), the second and third terms in the RHS are the $t > 0$ increment to marginal utility of leisure that accrues in the future because of n_2's role in assisting both research effort and increasing output. However, because n_2 has a direct and indirect effect (through production and investment specific technological change, respectively), the future discounted gains are adjusted by the term $[\gamma\theta-(1-\alpha)\xi(1-\gamma)]$. Going forward, it is important to note that if $[\gamma\theta-(1-\alpha)\xi(1-\gamma)] > 0$, then final good production is not n_2 intensive.

2.2.2 Decision Rules

We now derive the closed form decision rules based on the above first order conditions using the method of undetermined coefficients, as shown the following Lemma (1).

Lemma 1 *C_t, $I_t, n_t, n_{1t,}n_{2t}$ are given by (14), (15), (16), where $0 < \Phi < 1$ is given by (17), and $0 < x < 1$ given by (18) is a constant.*

$$C_t = \Phi_P Y_t(1-\tau), I_t = (1-\Phi_P)Y_t(1-\tau) \tag{14}$$

$$n_t = n_P = \frac{(1-\alpha)[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)]}{(1-\alpha)[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)]+\Phi_P x_P(1-\beta\gamma)}, \tag{15}$$

$$n_{1P} = x_P n_P, n_{2P} = (1-x_P)n_P, \tag{16}$$

where Φ_P is given by

$$\Phi_P = 1 - \frac{\alpha\beta(1-\beta\gamma)}{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)+\alpha\beta^3(1-\gamma)}, \tag{17}$$

and x_P is given by

$$x_P = \frac{(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}}{(1+\xi)(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}+\beta\theta(1-\Phi_P)}. \tag{18}$$

Proof. See Appendix A for derivations. ■

While decision rules for consumption and investment given by (14) suggest that levels of consumption and investment would fall if the proportional tax rate τ increases, the share of after tax income spent on consumption given by Φ_P increases when μ rises, and thereby for investment it falls. Intuitively, when μ rises the weight on the ratio of public capital to output, $\frac{G_t}{Y_{t-1}}$ in augmenting investment specific technological change increases and so the weight on the ratio $\frac{K_t}{Y_{t-1}}$ falls. Since the planner solves the optimization problem for the representative agent, the effect of increases in μ on private investments is therefore expected.

The labor supply is affected by μ. In fact, increases in μ has an ambiguous effect on n_{1P} but has a strong negative effect on n_{2P} which leads to an overall reduction in n_P.

An increase in μ increases the share of n_P devoted to n_{1P}, i.e., $\frac{\partial x_P}{\partial \mu} > 0$. Since $\frac{\partial \Phi_P}{\partial \mu} > 0$ from before, this implies $\frac{\partial n_P}{\partial \mu} < 0$.[19] To see this, we can decompose the total change in n_P

[19] See Appendix C

because of changes in μ by

$$\frac{\partial n_P}{\partial \mu} = \frac{\partial n_{1P}}{\partial \mu} + \frac{\partial n_{2P}}{\partial \mu}.$$

Given $\frac{\partial x_P}{\partial \mu} > 0$ and $\frac{\partial \Phi_P}{\partial \mu} > 0$ (and hence, $\frac{\partial (1-x_P)}{\partial \mu} < 0$) $\frac{\partial n_{2P}}{\partial \mu} < 0$ will be true. Since the change in n_{1P} due to a change in μ can be written as

$$\frac{\partial n_{1P}}{\partial \mu} = x_P \underbrace{\frac{\partial n_P}{\partial \mu}}_{<0} + n_P \underbrace{\frac{\partial x_P}{\partial \mu}}_{>0},$$

$\frac{\partial n_{1P}}{\partial \mu}$ may or may not be negative. Hence, while an increase in μ has an ambiguous effect on n_{1P}, it reduces n_{2P} and since the latter effect dominates, n_P falls. This implies that an increased weight of public capital induces agents to supply lesser labor particularly towards research effort (n_{2P}).

In contrast, an increase in ξ leads to an unambiguous increase in n_P (from (15)). This is because a rise in ξ leads to an increase in both n_P and $1 - x_P$. This implies that n_{2P} rises. However, the effect of an increase in ξ lowers n_{1P} unambiguously. Even though a rise in ξ leads to opposite effects on n_{1P} and n_{2P}, total labor supply, n_P, rises (see (16)).

2.2.3 Balanced Growth Path

We can easily obtain the balanced growth path (BGP) by substituting the above decision rules into the law of motion for investment specific technological change, (7). Define $\widehat{M_P}$ a constant as

$$\widehat{M_P} = B((1 - x_P)n_P)^{\theta}(1 - \Phi_P)^{(1-\mu)(1-\gamma)}. \tag{19}$$

Given the assumptions it is easy to show that we can obtain a constant growth rate for Z, K, G and Y. This condition necessarily implies $0 < \Phi_P$, x_P, $n_P < 1$ which always holds true. We therefore have the following Lemma (2).

Lemma 2 *On the steady state balanced growth path, the gross growth rate of Z, K, G and Y are given by (20), and (21)*[20]

$$\widehat{g_{z_P}} = [\widehat{M_P}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}} \tag{20}$$

$$\widehat{g_{k_P}} = \widehat{g_{g_P}} = \widehat{g_{z_P}}^{\frac{1}{1-\alpha}}, \widehat{g_{y_P}} = \widehat{g_{k_P}}^{\alpha} = \widehat{g_{z_P}}^{\frac{\alpha}{1-\alpha}}. \tag{21}$$

There are several aspects of the equilibrium growth rate worth mentioning. First, the growth rate is independent of the technology parameter, $\underline{A}$, as in Huffman (2008). Second,

[20] See Bishnu, Ghate and Gopalakrishnan (2011).

the growth rate of output, $\widehat{g_{y_P}}$, is less than $\widehat{g_{k_P}}$ along the balanced growth path because equation (7) is homogenous of degree $1+\theta$. Lemma (2) therefore clearly establishes that the effect of the stock of public capital on Z affects not just marginal productivity of factor inputs but also growth rate at the balanced growth path.

Finally, from (20), the tax rate exerts a positive effect on growth as well as a negative effect. This is similar to the equation characterizing the growth maximizing tax rate in models with public capital. The mechanism here is however different. For small values of the tax rate, a rise in τ leads to higher public capital relative to output, Y_{t-1}. This raises the future value of investment specific technological change, Z. An increase in Z reduces the real price of capital, stimulating investment and long run growth. However, for higher tax rates, further increases in the tax rate depresses after tax income, and investment. This reduces G relative to Y, lowering Z, and depressing investment and long run growth. Hence, there is a unique growth maximizing tax rate.

Using the expression for g_{z_P} in (20) we can characterize the growth maximizing tax rate as follows:

Proposition 1 *In the steady state, the planner maximizes growth by choosing the proportional tax rate given by $\tau = \mu$.*
Proof. *See Appendix A.* ■

Proposition (1) sets a benchmark for the planner to set the optimal tax rate. If the planner wants to maximize balanced growth, he should set the tax rate to μ. The higher the weight attached to $\frac{G_t}{Y_{t-1}}$ in the investment specific technological change equation, the higher should be the optimal tax rate set by the planner. This result is intuitive since it suggests that the government would have to impose a higher tax rate on income if public capital were to play a greater role in driving investment specific technological change.

2.2.4 Comparative Statics.

Equation (20) suggests that the equilibrium growth rate can be decomposed into two sources - those coming from the term,$\widehat{M_P}$ which captures the effects on growth from n_P, x_P, and Φ_P (terms that are independent of the proportional tax rate τ) and a composite (bracketed) term which captures the trade-offs of increasing the proportional tax:

$$\widehat{g_{z_P}} = \{\widehat{M_P}\}^{\frac{1}{2-\gamma}} \underbrace{[\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]}_{\text{The effects on growth from taxes}}{}^{\frac{1}{2-\gamma}}.$$

It is important to note that the characterization of optimal growth in the planning problem is identical to Barro (1990) as in Proposition (1). This is because along the balanced growth path, the growth rate is purely dependent on the growth rate of Z_t. But since public capital affects ISTC, it affects growth through the tax rate.

What happens to growth because of a change in the deep parameter μ? In particular, we choose two values of $\mu = \{0.5, 0.7\}$. Given the other parameter values,[21] figure (4) shows that an increase in μ from 0.5 to 0.7 increases the growth maximizing tax rate, which is expected, as seen in proposition (1). The plot shifts upward and skews to the right because an increase in μ from 0.5 to 0.7 reduces the optimal allocation towards n_2 which leads to a reduction in the growth rate for initially lower values of τ. However, for higher values of τ the contribution from $\frac{G_t}{Y_{t-1}}$ starts dominating and therefore, the growth rates are higher as compared to the growth rates for a lower value of μ.[22]

[Insert Figure 4]

2.3 The Competitive Decentralized Equilibrium

We now solve the competitive decentralized equilibrium. Consider an economy that is populated by a set of homogenous and infinitely lived agents. There is no population growth and the representative firms are completely owned by agents, who supply labor for final goods production, n_1, and R&D, n_2. Firms pay taxes (or receive subsidies) on capital income $\tau_k \in (-1, 1)$ while agents pay taxes (or receive subsidies) on labor income $\tau_n \in (-1, 1)$. Agents derive utility from consumption of the final good and leisure given in (1). The wage payment w_t for both kinds of labor are the same since there is no skill difference assumed between both activities. Agents fund consumption and investment decisions from their after tax wages which they receive for supplying labor n_1 and n_2, and capital income earned from holding assets, which essentially equals the returns to capital lent out for production at each time period t .

The representative firm produces the final good based on (3) but takes the externality from n_2 (given by $\overline{n}_2$) as given. Hence, the production function is given by

$$Y_t = \underbrace{\underline{A}\left(\overline{n}_{2t}^{1-\alpha}\right)^{\xi}}_{\text{Externality}} K_t^{\alpha} n_{1t}^{1-\alpha}$$

where the law of motion of private capital is given by (4). What is different compared to (3) is that the agent takes the externality due to n_2 as given. As mentioned earlier, agents also

[21] The value of other parameters are as follows: $\alpha = 0.35$, $\beta = 0.95$, $\gamma = 0.5$, $\delta = 1$, $\theta = 0.2$, $\xi = 1$. These parameter values - except for $\gamma-$ are borrowed from Huffman (2008), except for $\xi = 1$ which is the externality parameter due to n_{2_P} in our framework. We also choose the value of $B = 2$, which is the scaling parameter in Z.

[22] These results are however sensitive to level of persistence parameter γ.

treat the effect of $\frac{G_t}{Y_{t-1}}$ and $\frac{K_t}{Y_{t-1}}$ on Z_{t+1} as given. The government funds public investment, I_t^g, at each time period t using a distortionary tax imposed on labor, $\tau_n \in (-1,1)$, and capital, $\tau_k \in (-1,1)$ respectively. The following is therefore the government budget constraint:

$$I_t^g = w_t(n_{1t}+n_{2t})\tau_n + \{Y_t - w_t(n_{1t}+n_{2t})\}\tau_k.$$

Like Huffman (2008), we assume that profits are taxed according to the same rate as capital income.

2.3.1 The Firm's Dynamic Profit Maximization Problem

Firms solve their dynamic profit maximization problems which, at time t, have capital stock, K_t, and Z_t. Let $v(K_t, Z_t)$ denote the value function of the firm at time t. The returns to investment in the credit markets are given by r_t at time period t. The following is the firm's value function

$$v(K_t, Z_t) = \max_{K_{t+1}, n_{1t}, n_{2t}} \left\{ (Y_t - w_{1t}n_{1t} - w_{2t}n_{2t})(1-\tau_k) - \frac{K_{t+1}}{Z_t} + \frac{1}{1+r_t} v(K_{t+1}, Z_{t+1}) \right\}, \tag{22}$$

which it maximizes subject to (5) and (7).

The firm's maximization exercise yields the competitive factor prices for wages, and the return to capital. We therefore get the following first order conditions,[23]

$$\{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1+r_t}\right) \frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}$$

$$\{n_{1t}\} : w_t = \frac{(1-\alpha)Y_t}{n_{1t}}$$

$$\{n_{2t}\} : w_t(1-\tau_k) = \left(\frac{\theta}{n_{2t}}\right) \sum_{j=0}^{\infty} \gamma^j \left[\prod_{k=0}^{j} \frac{1}{1+r_{t+k}}\right] I_{t+j+1}.$$

In deriving these factor prices, we assume that the externality from n_2 in production is assumed to be given.

[23] See Appendix B.

2.3.2 The Agents Problem

Agents are allowed to borrow and lend by participating in the credit market. A representative agent maximizes (1) subject to the consumer budget constraint,

$$a_{t+1} = (1 + r_t)a_t + w_t(n_{1t} + n_{2t})(1 - \tau_n) - c_t, \tag{23}$$

the laws of motion given by (4), (5) and (7), total labor supply given by (2), and takes factor prices and profits as given. Agents therefore hold asset a_t which in equilibrium equals private capital accumulation used in production, as follows

$$a_t = K_t, \ \forall t \geq 0.$$

2.3.3 First Order Conditions

The following is the Lagrangian for the agent.

$$L = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_{1t} - n_{2t}) + \lambda_t \{(1 + r_t)a_t + w_t(n_{1t} + n_{2t})(1 - \tau_n) - c_t - a_{t+1}\}]. \tag{24}$$

The optimization conditions with respect to C_t, K_{t+1}, n_{1t}, and n_{2t}, are given by equations (25), (26), (27) and (28) respectively:

$$\{C_t\} : \tfrac{1}{C_t} = \lambda_t \tag{25}$$

$$\{a_{t+1}\} : \frac{\beta(1 + r_t)}{c_{t+1}} = \frac{1}{c_t} \tag{26}$$

$$\{n_{1t}\} : \frac{w_t(1 - \tau_n)}{c_t} = \frac{1}{1 - n_t} \tag{27}$$

$$\{n_{2t}\} : \frac{w_t(1 - \tau_n)}{c_t} = \frac{1}{1 - n_t}. \tag{28}$$

Once we substitute for factor prices from the firm's problem into equations (25), (26), (27) and (28), we obtain the following first order conditions for the competitive equilibrium:

$$\{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha \beta Y_{t+1}(1 - \tau_k)}{c_{t+1} K_{t+1}} \tag{29}$$

$$\{n_{1t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha) Y_t (1 - \tau_n)}{c_t n_{1t}} \tag{30}$$

$$\{n_{2t}\} : \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1-\tau_n}{1-\tau_k}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}. \tag{31}$$

Equation (29) is the standard Euler equation for the household. Compared to equation (11) in the planner's problem, the effect of the stock-externalities because of K and G on the inter-temporal savings decision is absent. This is because agents do not internalize this externality. Equations (30) and (31) equate the after tax wage to the MRS between consumption and leisure. Compared to equations (12) and (13) respectively, the additional terms due to the externalities are also absent.

2.3.4 Decision Rules

Based on the above first order conditions, the following Lemma (3) states the optimal decision rules for the agents.

Lemma 3 *C_t, $I_t, n_t, n_{1t,} n_{2t}$ are given by (32), (33), (34), where $0 < \Phi < 1$ is given by (35), and $0 < x < 1$ given by (36) is a constant.*

$$C_t = \Phi_{CE} A Y_t, I_t = (1-\Phi_{CE}) A Y_t \tag{32}$$

$$\textit{where, } A = \alpha(1-\tau_k) + (1-\alpha)(1-\tau_n) - \frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}$$

$$n_t = n_{CE} = \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n) + x_{CE}\Phi_{CE}A}, \tag{33}$$

$$n_{1CE} = x_{CE} n_{CE}, n_{2CE} = (1-x_{CE}) n_{CE}, \tag{34}$$

where Φ_{CE} is given by

$$\Phi_{CE} = 1 - \frac{\alpha\beta(1-\tau_k)}{A}, \tag{35}$$

and x_{CE} is given by

$$x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta + (1-\alpha)(1-\beta\gamma)}. \tag{36}$$

Proof. See Appendix B for details. ■

The above decision rules imply that depending upon the parameter values, there exists a feasible range of values that τ_k and τ_n can take such that

$$0 < A, \Phi_{CE}, n_{CE} < 1,$$

are true. The relationship between growth rates at the balanced growth path for private capital, public capital, output and investment specific technological change are identical to that for the planner's version, as given in Lemma (2).

2.4 Decentralizing the Planner's Growth Rate

We would like to ascertain under what conditions the competitive equilibrium allocations implement the planner's growth rate. We consider two cases: the case in which planner imposes equal factor income taxes on agents, i.e., $\tau_n = \tau_k = \tau$, and the case under which factor income taxes are unequal $\tau_n \neq \tau_k$.

2.4.1 Equal factor income taxes:

No externalities Suppose there are no externalities in the model, i.e., $\gamma = 1$ and as $\xi = 0$. Further, the government imposes equal factor income taxes on both capital and labor income, such that

$$\tau_n = \tau_k = \tau.$$

We show in Appendix C that equal factor income taxes will implement the planner's growth rate. In general, in the absence of the externalities, $\tau_k = \tau_n = \tau$, is the only factor income tax combination that implements the planner's growth rate.[24]

Externalities In this case (when $0 < \gamma < 1$ and $\xi > 0$) , the decision rules for the competitive equilibrium at optimum. C_t, $I_t, n_t, n_{1t,} n_{2t}$ are now given by (37), (38), (39), where $0 < \Phi_{CE} < 1$ is given by (40), and $0 < x_{CE} < 1$ given by (41) is a constant.

$$C_t = \Phi_{CE} A Y_t, I_t = (1 - \Phi_{CE}) A Y_t \tag{37}$$

$$\text{where, } A = (1 - \tau) \tag{38}$$

$$n_t = n_{CE} = \frac{(1 - \alpha)}{(1 - \alpha) + x_{CE} \Phi_{CE}}, \tag{39}$$

where Φ_{CE} is given by

$$\Phi_{CE} = 1 - \alpha\beta, \tag{40}$$

[24] This result also holds true when $\theta = 0$. In Appendix C we show that, under a knife-edge restriction, $\left(\frac{1-\beta}{\beta}\right)^2 = \theta$, any factor income tax combination (including the case of equal factor income taxes) implements the planner's growth rate.

and x_{CE} is given by

$$x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta + (1-\alpha)(1-\beta\gamma)}. \tag{41}$$

When factor income taxes are equal, growth rates in the competitive environment is maximized when $\tau = \mu$.[25] However, since agents do not internalize the externality in both production and investment specific technological change, the competitive equilibrium growth rate may not be equal to the planner's growth rate. However, as the level of persistence, γ (the coefficient on Z), in investment specific technological change increases, and as the externality in production due to the choice n_{2CE} decreases (i.e., the effect of all three externalities diminish) the decision rules for the agent coincide with that of the planner and hence growth rates coincide.

2.4.2 Unequal factor income taxes: a simple numerical exercise

In this section, we conduct a simple numerical exercise to match the calibrated factor income tax gaps from our model with the tax gaps identified in Figures (1) and (2).[26] As noted in the introduction, the factor income tax mix that implements the planner's growth rate is indeterminate because the planner's growth maximizing allocations are implemented with a fixed tax rate on output. Our strategy is to first fix the tax on labor income, τ_n. For a fixed labor income tax rate, we then calibrate the tax on capital income that decentralizes the planner's allocations numerically. We show that we can restore the planner's growth rate by varying the production externalities so that the tax gaps match – up to a reasonable approximation – those observed in the data.

Since empirical magnitudes of externalities associated with the stock of public and private capital, and specialized research input, are not available in the literature, we focus our results on checking whether large factor income tax gaps of the magnitude reported in the introduction can be generated by small changes in the size of externalities. We assume that the externality parameter governing how much labor devoted to improving capital quality affects productivity (ξ) – is small. We therefore fix the value of $\xi = 0.1$. In contrast, the stock externalities – captured by the term $(1-\gamma)$ – relate to public and private capital stocks in national economies, which can be large or small. We therefore vary γ from 0 to 1. Our other parameters are given by: $\alpha = 0.35, \beta = 0.95, \delta = 1, \theta = 0.2$ which are taken from Huffman (2008). Parameters $\underline{A}$ and B are constant scaling parameters: we arbitrarily choose them to be $\underline{A} = 1$ and $B = 4$. We consider the value of $\mu = 0.5$ to allow for both public and private capital externalities to have equal weight. These parameters allow us to

[25] The proof of this is similar to the proof of Proposition (1) which can be refered to in Appendix A.

[26] For our numerical experiments we assume that $\{\tau_n, \tau_k\} \in [0, 1)$.

compute the planner's growth rate from equation (20).

We define the iso-growth loci as representing all combinations of factor income taxes that implement the planner's growth rate. Figure (5) plots the iso-growth loci where each upward sloping locus represents the planner's growth rate for a specific value of the parameter, γ, where all the other parameters are unchanged. Figure (5) illustrates two results.

[Insert Figure (5)]

First, both factor income tax combinations converge towards equality as the magnitude of externalities diminish. That is, for a given level of ξ, as the value of $\gamma \rightarrow 1$, the iso-growth locus for (τ_n, τ_k) shifts up and approximates the 45˙ line from below. This means for a given tax on labor income, τ_n, with higher spillovers from G and K (i.e., $\gamma \rightarrow 0$) , a lower τ_k implements the planner's allocations because under-accumulation of capital is high. That is, factor income taxes diverge when the spillovers are large.

Second, Figure (5) show that as γ increases (and converges to 1), the implied factor income tax gaps are roughly within the range observed in the data. As pointed out in the introduction, 12 out of 17 OECD economies have a tax on labor income greater than a tax on capital income (from Figure 1 and 2). The average tax gap for these 12 economies is roughly 0.136, where the $\pm$ two standard deviation yields an interval of $(0.01, 0.27)$. To generate tax gaps in this range, we assume that τ_n is greater than τ_k. We then calibrate τ_k by varying γ from 0 to 1 (with $\xi = 0.1$). For example, in Figure (5), point (a) is on an iso-growth locus that assumes $\gamma = 0.9$ and $\tau_n = 0.5$. The corresponding calibrated tax on capital income is 0.21 which yields a tax gap $(\tau_n - \tau_k)$ of 0.29, marginally higher than the average tax gap observed in, say, a country like Austria (which is 0.26). Point (b) is on an iso-growth locus that assumes $\gamma = 0.95$, where again $\tau_n = 0.5$. The calibrated tax on capital income is $\tau_k = 0.31$ which yields a tax gap $(\tau_n - \tau_k)$ of 0.19 roughly identical to the average factor income tax gap observed in, say, Finland. Finally, point (c) corresponds to the case when $\gamma = 1$. This shows that $\tau_n - \tau_k \neq 0$ since $\xi \neq 0$.If $\xi = 0$, the iso-growth locus converges to the 45 degree line, and $\tau_n - \tau_k = 0$.Intuitively, when agents don't internalize the role that public and private capital aggregates have on ISTC, they under-invest in capital. The stronger the magnitude of the externalities, the higher is the extent of the under-investment, and so the larger is the factor income tax gap. Therefore, the planner's growth rate can be restored by taxing capital income at a lower rate. Thus, our numerical experiments confirm that small values of externalities $(1 - \gamma, \xi)$ yield large factor income tax gaps when the planner's growth rate is implemented.[27] We summarize this in terms of the following remark:

[27] However, there exist parameter combinations under which the ranking on factor income tax levels can get reversed. This happens when α is high. This implies that the ranking between τ_k and τ_n is sensitive to

Remark 1 *The presence of externalities gives rise to unequal factor income tax rates that implement the planner's growth rate. As the magnitude of externalities diminish, these factor income tax rates converge towards equality. Numerically, we find that small values of externalities* $(1-\gamma, \xi)$ *can yield large factor income tax gaps when the planner's growth rate is implemented.*

Empirically, equal factor income taxes are rarely observed in the data. Our claim is that these externalities matter in explaining factor income tax differences in advanced economies. There are two aspects that should be noted. As shown in Figure (2) similarly growing economies factor income taxes are not just unequal, but the absolute gaps between the two also vary. As shown in Figure (1 and 2), there is no clear ranking between the two level of factor income tax rates although in general, $\tau_n > \tau_k$. By incorporating different production externalities in a model of endogenous investment specific technological change, our results yield this outcome. More generally, we show that different parametric values for these externalities can help explain factor income tax gaps that we observe in actual economies.

2.5 Welfare

We assume that the values of K_0 and G_0 are such that the economy is on the balanced growth path. Given this, we compute welfare for agents by substituting the representative agent's optimal decision rules given in Lemma (3) and given by (32), (33), (34), (35), and (36) into the representative agent's discounted life time utility function given by (1). This yields the following expression[28]

$$\Lambda = \frac{\log[\Phi_{CE}]}{1-\beta} + \frac{\log[Y_0]}{1-\beta} + \frac{\log[A(\tau_k, \tau_n)]}{1-\beta} + \frac{\beta^2\alpha}{(1-\beta)(1-\alpha)} \log g_{c,CE} + \frac{\log(1-n_{CE})}{1-\beta}. \quad (42)$$

We then ask how a factor income tax combination that maximizes welfare compares with the factor income tax rates (τ_n, τ_k) that restores the planner's growth rate. Our result – which we are only able to show numerically – is that different magnitudes of the key externality parameters – γ and ξ – influence the welfare maximizing factor income tax combinations. We assume γ takes arbitrary values $\{0.3, 0.9\}$ that is, a high externality and low externality case; meanwhile, we fix $\xi = 0.1$.

factor shares in final good production. For instance, if $\gamma = 0.4; \mu = 0.6; \theta = 0.5; \alpha = 0.7$, $\tau_k > \tau_n$. Finally, the optimal tax on capital can also be a subsidy$(\tau_k < 0)$, so that the planner can restore the equilibrium growth rate by subsidizing capital income. For instance, this obtains when γ is low and θ is high.

[28]See Devarajan et al. (1998). For the entire welfare calculation, see Appendix D.

As in Figure (5), in Figure (6) we plot the iso-growth locus when the externality due to n_2 in production is marginal ($\xi = 0.1$). This locus represents all factor income tax combinations as in Figure (5) that implements the planner's growth rate. The welfare maximizing tax combination - which is unique - is indicated by the circle in Figure (6), which is underneath this iso-growth locus. This means that for the welfare maximizing tax to replicate the planner's growth rate, the tax on capital income needs to be higher. The result is similar when we have higher values of γ.[29] As can be seen in Figure (6) the welfare maximizing tax on capital income is always less than the labor income tax rate. Intuitively, because of strong production externalities, there is under-accumulation of capital. Therefore, in order to get the planner to get to the iso-growth locus, the tax on capital income needs to be lower.

[Insert Figure 6]

We generalize these results in terms of the following remark:

Remark 2 *When there are no externalities* ($\xi = 0, \gamma = 1$), *and investment specific technological change is exogenous* ($\theta = 0$), *the unique welfare maximizing tax combination replicates the planner's growth rate. This happens because from equation (20),* $g_Z = B$ *for the planner and from the decentralized equilibrium. However, when investment specific technological change is endogenous* ($\theta \neq 0$) *and there are externalities* ($\xi > 0, 0 < \gamma < 1$), *then the iso-growth locus of factor income tax combinations always yields lower welfare. Therefore, the presence of both production externalities and endogenous ISTC imply departures from the factor income tax mix that implement the planner's growth rate.*

The above remark suggests that the departure of the welfare maximizing tax rate from the iso-growth locus has two sources 1) the effect of externalities and 2) the effect of n_2 on production and ISTC. When production externalities are absent ($\xi = 0, \gamma = 1$), and ISTC is endogenous ($\theta \neq 0$), the welfare maximizing tax mix *does not* coincide with the iso-growth locus. The lower tax on capital income relative to the tax on capital income obtained in this case is because of the role that endogenous ISTC has on capital accumulation. With the additional restriction that $\theta = 0$, ISTC becomes exogenous, and the welfare maximizing tax mix coincides with the iso-growth locus. Therefore, both production externalities and endogenous ISTC imply departures from the factor income tax mix that restores the planner's growth rate.

[29] We have not explicitly presented the case where there is a high externality due to n_2 in production ($\xi = 1$). We can show that when γ takes on a low value of 0.3 the iso-growth locus will be distinctly below the unique welfare maximizing tax rate. This changes when γ is high and is equals to 0.9.

3 Conclusion

This paper constructs a simple and tractable endogenous growth model with spillovers from the stock of public and private capital which influence investment specific technological change. We focus on the steady state balanced growth path. Our model is motivated by the empirical observation that advanced economies experiencing similar or identical growth rates have widely varying factor income tax combinations. We characterize the planner's problem and show that the constrained first best fiscal policy yields an indeterminate combination of capital tax rates and the labor tax rates. This allows us to quantify and discuss intuitively *how specific externalities* can have a bearing on the trade-off between the optimal factor income tax mix. In the welfare analysis, our framework allows us to also Pareto rank various combinations of factor income taxes that implement the planner's growth rate. We show that both endogenous investment specific technological change as well as the presence of the externalities imply deviations from the constrained first best tax mix. To the extent that such spillovers exist in actual economies, they have a bearing on the optimal factor income tax mix that implements the constrained first best optimum on a balanced growth path.

While we do not solve for the Ramsey allocations (second best fiscal policy), our results are closely related to a celebrated literature started by Judd (1985) and Chamley (1986) who find that capital taxation decreases welfare and a zero capital tax is thus efficient in the long-run steady state. From a growth standpoint, models analyzing the equilibrium relationship between capital income taxes and growth also typically find that an increase of the capital income tax reduces the return to private investment, which in turn implies a decrease of capital accumulation and thus growth (see Lucas (1990) and Rebelo (1991)). In contrast, our results are consistent with some other papers in this literature which show that the optimal capital income tax is positive, i.e., high capital income taxation may restore the planner's growth rate (see Uhlig and Yanagawa (1996) and Rivas (2003)). In terms of future work, one could formalize the second best Ramsey policy within our environment.

Future work can extend our framework to think about comparing the growth and welfare effects of optimal tax policy on research and development versus funding public investment. In addition, our model characterizes the optimal tax rate along the balanced growth path. Future work can model the transitional dynamics.

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Technical Appendix

Appendix A: The Planner's Version

$$\{C_t\} : \tfrac{1}{C_t} = \lambda_t,$$

$$\{K_{t+1}\} \quad : \quad \frac{-\lambda_t}{Z_t} + \beta\lambda_{t+1}\frac{\alpha Y_{t+1}(1-\tau)}{K_{t+1}} - \beta\lambda_{t+1}\frac{\partial}{\partial K_{t+1}}(\frac{K_{t+2}}{Z_{t+1}}) - \beta^2\lambda_{t+2}\frac{\partial}{\partial K_{t+1}}(\frac{K_{t+3}}{Z_{t+2}}) - ... = 0.$$

$$\Rightarrow \quad \frac{1}{C_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}} + \frac{\beta}{C_{t+1}}\frac{K_{t+2}}{Z_{t+1}^2}\frac{\partial Z_{t+1}}{\partial K_{t+1}} + \frac{\beta^2}{C_{t+2}}\frac{K_{t+3}}{Z_{t+2}^2}\frac{\partial Z_{t+2}}{\partial K_{t+1}} + ...$$

where,

$$\frac{\partial Z_t}{\partial K_{t+1}} = \frac{\partial Z_{t+1}}{\partial K_{t+1}} = 0,\ \frac{\partial Z_{t+2}}{\partial K_{t+1}} = (1-\gamma)(1-\mu)\frac{Z_{t+2}}{K_{t+1}},\ \frac{\partial Z_{t+3}}{\partial K_{t+1}} = \frac{\gamma Z_{t+3}}{Z_{t+2}}\frac{\partial Z_{t+2}}{\partial K_{t+1}} - \alpha(1-\gamma)\frac{Z_{t+3}}{K_{t+1}}.$$

$$\Rightarrow \frac{\partial Z_{t+3}}{\partial K_{t+1}} = (1-\gamma)\frac{Z_{t+3}}{K_{t+1}}(\gamma(1-\mu)-\alpha).$$

$$\Rightarrow \frac{\partial Z_{t+3+j}}{\partial K_{t+1}} = \gamma^j(1-\gamma)\frac{Z_{t+3+j}}{K_{t+1}}[\gamma(1-\mu)-\alpha],\ \text{for } j \geqq 0.$$

Hence,

$$\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}} + \beta^2\frac{I_{t+2}}{C_{t+2}}\frac{(1-\gamma)(1-\mu)}{K_{t+1}} + \frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha)}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{C_{t+j+3}} \tag{43}$$

The FOC with respect to n_{1t} is as follows.

$$\{n_{1t}\} : \frac{-1}{1-n_t} + \frac{\lambda_t(1-\alpha)Y_t(1-\tau)}{n_{1t}} - \lambda_t\frac{\partial}{\partial n_{1t}}(\frac{K_{t+1}}{Z_t}) - \beta\lambda_{t+1}\frac{\partial}{\partial n_{1t}}(\frac{K_{t+2}}{Z_{t+1}}) - \beta^2\lambda_{t+2}\frac{\partial}{\partial n_{1t}}(\frac{K_{t+3}}{Z_{t+2}}) - ... = 0$$

where

$$\frac{\partial Z_t}{\partial n_{1t}} = \frac{\partial Z_{t+1}}{\partial n_{1t}} = 0,\ \frac{\partial Z_{t+2}}{\partial n_{1t}} = -(1-\gamma)(1-\alpha)\frac{Z_{t+2}}{n_{1t}},\ \frac{\partial Z_{t+3}}{\partial n_{1t}} = \frac{\gamma Z_{t+3}}{Z_{t+2}}\frac{\partial Z_{t+2}}{\partial n_{1t}} \text{ and so on.}$$

Hence,

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau)}{C_t n_{1t}} - \frac{\beta^2(1-\alpha)(1-\gamma)}{n_{1t}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{C_{t+j+2}} \tag{44}$$

Similarly, the FOC with respect to n_{2t} is given by

$$\{n_{2t}\}: \frac{-1}{1-n_t}+\frac{\lambda_t(1-\alpha)\xi Y_t(1-\tau)}{n_{2t}}-\lambda_t\frac{\partial}{\partial n_{2t}}(\frac{K_{t+1}}{Z_t})-\beta\lambda_{t+1}\frac{\partial}{\partial n_{2t}}(\frac{K_{t+2}}{Z_{t+1}})-\beta^2\lambda_{t+2}\frac{\partial}{\partial n_{2t}}(\frac{K_{t+3}}{Z_{t+2}})...=0$$

where,

$$\begin{aligned}\frac{\partial Z_t}{\partial n_{2t}} &= 0,\ \frac{\partial Z_{t+1}}{\partial n_{2t}}=\frac{\theta Z_{t+1}}{n_{2t}};\\ \frac{\partial Z_{t+2}}{\partial n_{2t}} &= \frac{\gamma Z_{t+2}}{Z_{t+1}}\frac{\theta Z_{t+1}}{n_{2t}}-(1-\alpha)\xi(1-\gamma)\frac{Z_{t+2}}{n_{2t}}\end{aligned}$$

$$\Rightarrow \frac{\partial Z_{t+2}}{\partial n_{2t}}=(\gamma\theta-(1-\alpha)\xi(1-\gamma))\frac{Z_{t+2}}{n_{2t}}.$$

$$\Rightarrow \frac{\partial Z_{t+j+2}}{\partial n_{2t}}=\gamma^j(\gamma\theta-(1-\alpha)\xi(1-\gamma))\frac{Z_{t+j+2}}{n_{2t}},\quad \text{for } j\geqq 0.$$

Hence,

$$\{n_{2t}\}: \frac{1}{1-n_t}=\frac{(1-\alpha)\xi Y_t(1-\tau)}{C_t n_{2t}}+\beta\theta\frac{I_{t+1}}{C_{t+1}n_{2t}}+\frac{\beta^2(\gamma\theta-(1-\alpha)\xi(1-\gamma))}{n_{2t}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{C_{t+j+2}}. \tag{45}$$

The Decision Rules

We use the method of undetermined coefficients to solve out for the decision rules.

$$\begin{aligned}C_t &= \Phi_P Y_t(1-\tau),\\ I_t &= (1-\Phi_P)Y_t(1-\tau)\\ n_1 &= x_P n_P\\ n_2 &= (1-x_P)n_P\\ n_t &= \overline{n}.\end{aligned}$$

$$\{K_{t+1}\}: \frac{1}{C_t Z_t}=\frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}}+\beta^2\frac{I_{t+2}}{C_{t+2}}\frac{(1-\gamma)(1-\mu)}{K_{t+1}}+\frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha)}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{C_{t+j+3}}$$

This implies,

$$\Rightarrow \quad \frac{1}{\Phi_P Y_t(1-\tau)Z_t} = \beta\frac{\alpha Y_{t+1}(1-\tau)}{\Phi_P Y_{t+1}(1-\tau)(1-\Phi_P)Y_t(1-\tau)Z_t} + \beta^2\frac{(1-\Phi_P)}{\Phi_P(1-\Phi_P)Y_t(1-\tau)Z_t}(1-\gamma)(1-\mu)$$
$$+\frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha)}{(1-\Phi_P)Y_t(1-\tau)Z_t}(\frac{1}{1-\beta\gamma})\frac{(1-\Phi_P)}{\Phi_P}$$

$$\Rightarrow (1-\Phi_P) = \frac{\alpha\beta(1-\beta\gamma)}{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha]}. \tag{46}$$

From the FOC for n_{1t},

$$\frac{n_P}{1-n_P} = \frac{(1-\alpha)}{\Phi_P x_P} - \frac{\beta^2(1-\alpha)(1-\gamma)(1-\Phi_P)}{x_P(1-\beta\gamma)\Phi_P}$$

$$\Rightarrow n_P = \frac{(1-\alpha)[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)]}{(1-\alpha)[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)]+\Phi_P x_P(1-\beta\gamma)}. \tag{47}$$

From the FOC $\{n_{2t}\}$

$$\{n_{2t}\}:(1-x_P)\left(\frac{n_P}{1-n_P}\right) = \frac{(1-\alpha)\xi}{\Phi_P}+\beta\theta\left(\frac{1-\Phi_P}{\Phi_P}\right)+\beta^2(\gamma\theta-(1-\alpha)\xi(1-\gamma))\left(\frac{1}{1-\beta\gamma}\right)\left(\frac{1-\Phi_P}{\Phi_P}\right)$$

$$\left(\frac{1-x_P}{x_P}\right) = \frac{(1-\alpha)\xi\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}+\beta\theta(1-\Phi_P)(1-\beta\gamma)+\beta^2\gamma\theta(1-\Phi_P)}{(1-\alpha)(1-\beta\gamma)-\beta^2(1-\alpha)(1-\gamma)(1-\Phi_P)}$$

Hence,

$$x_P = \frac{(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}}{(1+\xi)(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}+\beta\theta(1-\Phi_P)}. \tag{48}$$

As long as $0<(1-\Phi_P)<1$ and $(1-\beta\gamma)-\beta^2(1-\gamma)>0$, we can easily show $0<\Phi_P$, x_P, $n_P<1$. Note,

$$\begin{aligned}(1-\beta\gamma)-\beta^2(1-\gamma) &= 1-\beta\gamma-\beta^2+\beta^2\gamma \\ &= 1-\beta^2-\beta\gamma(1-\beta) \\ &= (1-\beta)[1+\beta-\beta\gamma],\end{aligned}$$

which is clearly positive as long as $0 < \gamma, \beta < 1$, which is assumed. Now,

$$\begin{aligned}(1-\Phi_P) &= \frac{\alpha\beta(1-\beta\gamma)}{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha]} \\ &= \frac{\alpha\beta(1-\beta\gamma)}{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)+\alpha\beta^3(1-\gamma)} > 0\end{aligned}$$

Since,

$$\beta^2(1-\gamma)[(1-\mu)-\alpha\beta] < (1-\alpha\beta)(1-\beta\gamma),$$

we get, $0 < \Phi_P$, x_P, $n_P < 1$.

Growth rate at the BGP

$$Y_t = \underline{A}.\left(n_{2t}^{1-\alpha}\right)^{\xi} K_t^{\alpha} n_{1t}^{1-\alpha}$$

At the balanced growth path (BGP),

$$g_{y_P} = g_{y_{Pt+1}} = \frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}^{\alpha}}{K_t^{\alpha}} = g_{k_{Pt+1}}^{\alpha} = g_{k_P}^{\alpha},$$

$$\text{and } g_{k_P} = \frac{K_{t+1}}{K_t} = \frac{I_t Z_t}{I_{t-1}Z_{t-1}} = g_{y_P}.g_{z_P}.$$

Hence,

$$g_{y_P} = g_{z_P}^{\frac{\alpha}{1-\alpha}}, g_{k_P} = g_{g_P} = g_{z_P}^{\frac{1}{1-\alpha}}.$$

Proposition (1)

$$\widehat{g_{z_P}} = [\widehat{M_P}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}},$$

$$\frac{\partial\widehat{g_{z_P}}}{\partial\tau} = 0,$$

$$\begin{aligned}&\Rightarrow \quad \mu(\tau)^{\mu-1}(1-\tau)^{1-\mu} - (1-\mu)(\tau)^{\mu}(1-\tau)^{-\mu} = 0 \\ &\Rightarrow \quad \tau = \mu.\end{aligned}$$

Appendix B: Agent's Version

$$\{K_{t+1}\} : \frac{-1}{Z_t} + \left(\frac{1}{1+r}\right)\frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}} = 0.$$

$$\Rightarrow \{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1+r}\right) \frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}. \tag{49}$$

$$\{n_{1t}\} : \frac{(1-\alpha)Y_t(1-\tau_k)}{n_{1t}} - w_t(1-\tau_k) = 0$$

$$\Rightarrow \{n_{1t}\} : w_t = \frac{(1-\alpha)Y_t}{n_{1t}}. \tag{50}$$

Finally,

$$\{n_{2t}\} : w_t(1-\tau_k) = \left(\frac{\theta}{n_{2t}}\right) \sum_{j=0}^{\infty} \gamma^j \left[\prod_{k=0}^{j} \frac{1}{1+r_{t+k}}\right] I_{t+j+1}. \tag{51}$$

The Consumer's Problem

$$\begin{aligned}
\{c_t\} &: \frac{1}{c_t} = \lambda_t, \\
\{a_{t+1}\} &: \frac{\beta(1+r)}{c_{t+1}} = \frac{1}{c_t} \\
\{n_{1t}\} &: \frac{w_t(1-\tau_n)}{c_t} = \frac{1}{1-n_t} \\
\{n_{2t}\} &: \frac{w_t(1-\tau_n)}{c_t} = \frac{1}{1-n_t}
\end{aligned}$$

From the firm's FOC $\{K_{t+1}\}$:

$$\{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1+r}\right) \frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}.$$

Substituting for $(1+r)$ from $\{a_{t+1}\}$

$$\begin{aligned}
\Rightarrow \quad & \frac{1}{Z_t} = \frac{\beta c_t}{c_{t+1}} \left[\frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}\right] \\
\Rightarrow \quad & \{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha \beta Y_{t+1}(1-\tau_k)}{c_{t+1} K_{t+1}}
\end{aligned}$$

Similarly,

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau_n)}{c_t n_{1t}}$$

and,

$$\{n_{2t}\} : \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right) \left(\frac{1-\tau_n}{1-\tau_k}\right) \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+1}}{c_{t+j+1}}.$$

To summarize all FOCs,

$$\begin{aligned}
\{K_{t+1}\} &: \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}} \\
\{n_{1t}\} &: \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau_n)}{c_t n_{1t}} \\
\{n_{2t}\} &: \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1-\tau_n}{1-\tau_k}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}.
\end{aligned}$$

When

$$\tau_k = \tau_k = \tau,$$

we have

$$\begin{aligned}
\{K_{t+1}\} &: \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{c_{t+1}K_{t+1}} \\
\{n_{1t}\} &: \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau)}{n_{1t}} \\
\{n_{2t}\} &: \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}.
\end{aligned}$$

The Decision Rules

We use the method of undetermined coefficients to obtain the decision rules

$$\begin{aligned}
C_t &= \Phi_{CE} A Y_t, \\
I_t &= (1-\Phi_{CE}) A Y_t \\
n_{1t} &= x_{CE} n_{CE} \\
n_{2t} &= (1-x_{CE}) n_{CE} \\
n_t &= n_{CE},
\end{aligned}$$

where,

$$\{Y_t - w_t(n_{1t}+n_{2t})\}(1-\tau_k) + w_t(n_{1t}+n_{2t})(1-\tau_n) = AY_t.$$

$$\Rightarrow [\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)]Y_t + w_t n_{2t}(\tau_k-\tau_n) = AY_t$$

$$\Rightarrow [\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)]Y_t + \left\{\frac{\beta\theta A Y_t\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}\right\}(\tau_k-\tau_n) = AY_t$$

$$\Rightarrow \alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)+\frac{\beta\theta A\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)=A$$

$$\Rightarrow Y_t\left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)+\frac{\beta\theta A\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right]=AY_t,$$

$$\Rightarrow A=\left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)+\frac{\beta\theta\,(1-\Phi)\,A}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right]. \tag{52}$$

From the FOC of $\{K_{t+1}\}$

$$\{K_{t+1}\}:\frac{1}{c_tZ_t}=\frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}}$$

This implies,

$$\frac{1}{\Phi_{CE}AY_tZ_t}=\frac{\alpha\beta Y_{t+1}(1-\tau_k)}{\Phi AY_{t+1}(1-\Phi_{CE})AY_tZ_t}$$

$$\Rightarrow (1-\Phi_{CE})=\frac{\alpha\beta(1-\tau_k)}{A}. \tag{53}$$

Substituting for $(1-\Phi_{CE})A$ from 53 into 52,

$$\begin{aligned}\Rightarrow\quad A &= \left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)+\frac{\beta\theta\,(1-\Phi_{CE})\,A}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right] \qquad (54)\\ &= \alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)-\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}.\end{aligned}$$

When $\tau_n=\tau_k=\tau$

$$\begin{aligned}A &= [\alpha(1-\tau)+(1-\alpha)(1-\tau)]\\ &= (1-\tau).\end{aligned}$$

From $\{n_{1t}\}$ we get

$$\begin{aligned}\{n_{1t}\}\quad : \quad \frac{x_{CE}n_{CE}}{1-n_{CE}} &= \frac{(1-\alpha)Y_t(1-\tau_n)}{\Phi_{CE}AY_t}\\ \Rightarrow\quad \frac{x_{CE}n_{CE}}{1-n_{CE}} &= \frac{(1-\alpha)(1-\tau_n)}{\Phi_{CE}A}\\ \Rightarrow\quad \frac{n_{CE}}{1-n_{CE}} &= \frac{(1-\alpha)(1-\tau_n)}{x_{CE}\Phi_{CE}A}\end{aligned}$$

$$\Rightarrow n_{CE}=\frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n)+x_{CE}\Phi_{CE}A}. \tag{55}$$

From $\{n_{2t}\}$

$$\{n_{2t}\}: \frac{(1-x)n_{CE}}{1-n_{CE}} = \frac{\beta\theta}{(1-\beta\gamma)}\left(\frac{1-\tau_n}{1-\tau_k}\right)\frac{(1-\Phi_{CE})}{\Phi_{CE}}$$

$$\Rightarrow \frac{(1-\alpha)(1-\tau_n)}{\Phi_{CE}A}\frac{(1-x_{CE})}{x_{CE}} = \frac{\beta\theta}{(1-\beta\gamma)}\left(\frac{1-\tau_n}{1-\tau_k}\right)\frac{(1-\Phi_{CE})}{\Phi_{CE}}$$

$$\Rightarrow \frac{(1-x_{CE})}{x_{CE}} = \frac{A\beta\theta(1-\Phi_{CE})}{(1-\alpha)(1-\beta\gamma)(1-\tau_k)}.$$

$$\Rightarrow x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)(1-\tau_k)}{A\beta\theta(1-\Phi_{CE})+(1-\alpha)(1-\tau_k)(1-\beta\gamma)}. \tag{56}$$

Since,

$$A(1-\Phi_{CE}) = \alpha\beta(1-\tau_k),$$

$$\Rightarrow x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)}.$$

Appendix C: Equal factor income taxes

$$(1-\Phi_P) = \frac{\alpha\beta(1-\beta\gamma)}{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)+\alpha\beta^3(1-\gamma)}.$$

As μ increases, $(1-\mu)$ decreases, which implies $-(1-\mu)$ in the denominator increases and therefore $(1-\Phi_P)$ declines.

$$\Rightarrow \frac{\partial\Phi_P}{\partial\mu} > 0$$

We will now look at x_P :

$$\begin{aligned}\frac{1}{x_P} &= (1+\xi) + \frac{\beta\theta(1-\Phi_P)}{(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}}\\ &= (1+\xi) + \frac{\alpha\beta^2\theta}{(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)\}}.\end{aligned}$$

As μ increases the term $\frac{\alpha\beta^2\theta}{(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)\}}$ declines.

$$\Rightarrow \frac{\partial x_P}{\partial\mu} > 0.$$

We will now look at n_P :

$$\begin{aligned}
\frac{1}{n_P} &= 1+\frac{x_P\Phi_P(1-\beta\gamma)}{(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}}\\
&= 1+x_P\frac{\Phi_P(1-\beta\gamma)}{(1-\alpha)\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}}\\
&= 1+\frac{x_P}{(1-\alpha)}\left[\frac{\Phi_P(1-\beta\gamma)}{\{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}}\right]\\
&= 1+\frac{x_P}{(1-\alpha)}\left[1-\frac{\alpha\beta\left((1-\beta\gamma)-\beta^2(1-\gamma)\right)}{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)}\right]
\end{aligned}$$

We know that as μ increases, $\frac{x_P}{(1-\alpha)}$ increases because $\frac{\partial x_P}{\partial \mu}>0$. The term $\left[1-\frac{\alpha\beta\left((1-\beta\gamma)-\beta^2(1-\gamma)\right)}{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)}\right]$ also increases as μ increases. Hence

$$\Rightarrow \frac{\partial n_P}{\partial \mu}<0.$$

In the competitive equilibrium under equal factor income taxes,

$$\begin{aligned}
A &= 1-\tau.\\
&\Rightarrow (1-\Phi_{CE})=\alpha\beta\\
&\Rightarrow n_{CE}=\frac{(1-\alpha)}{(1-\alpha)+x_{CE}\Phi_{CE}}\\
&\Rightarrow x_{CE}=\frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)}.
\end{aligned}$$

$$\Rightarrow \frac{g_{z_P}}{g_{z_{CE}}}=\frac{\left((1-x_P)^\theta(n_P)^\theta(1-\Phi_P)^{(1-\mu)(1-\gamma)}\right)^{\frac{1}{2-\gamma}}}{\left((1-x_{CE})^\theta(n_{CE})^\theta(1-\Phi_{CE})^{(1-\mu)(1-\gamma)}\right)^{\frac{1}{2-\gamma}}}.$$

Φ_{CE} is independent of μ. However, since $0<\Phi_{CE}<1$,

$$\Rightarrow \frac{\partial(1-\Phi_{CE})^{(1-\mu)}}{\partial\mu}>0.$$

We know the term $(1-\Phi_P)$ is given by

$$(1-\Phi_P)=\frac{\alpha\beta(1-\beta\gamma)}{(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)+\alpha\beta^3(1-\gamma)}.$$

As $\gamma\rightarrow 1$,

$$1-\Phi_P\rightarrow 1-\alpha\beta=1-\Phi_{CE}.$$

Similarly, as $\gamma \to 1$ and as $\xi \to 0$,

$$\begin{aligned} x_P &\to x_{CE} \\ n_P &\to n_{CE}. \end{aligned}$$

$$\Rightarrow g_{z_{CE}} \to g_{z_P}.$$

The no externalities case

Suppose $\gamma = 1$ and $\xi = 0$. The FOC for the planner's version are then given by

$$\{C_t\} : \tfrac{1}{C_t} = \lambda_t$$

$$\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}}$$

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau)}{C_t n_{1t}}$$

$$\{n_{2t}\} : \frac{1}{1-n_t} = \frac{\beta\theta}{n_{2t}} \sum_{j=0}^{\infty} \beta^j \frac{I_{t+j+2}}{C_{t+j+2}}.$$

The FOCs for the agents are summarized as follows,

$$\begin{aligned} \{K_{t+1}\} &: \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}} \\ \{n_{1t}\} &: \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau_n)}{c_t n_{1t}} \\ \{n_{2t}\} &: \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1-\tau_n}{1-\tau_k}\right) \sum_{j=0}^{\infty} \beta^j \frac{I_{t+j+1}}{c_{t+j+1}}. \end{aligned}$$

The FOCs coincide when

$$\tau_n = \tau_k = \tau.$$

This implies, the optimal solutions always coincide for the planner and for the agent under equal factor income taxes. For the planner, under no externalities,

$$
\begin{aligned}
(1-\Phi_P) &= \alpha\beta \\
\Phi_P &= 1-\alpha\beta \\
n_P &= \frac{(1-\alpha)}{(1-\alpha)+\Phi_P x_P} \\
x_P &= \frac{(1-\alpha)(1-\beta)}{(1-\alpha)(1-\beta)+\alpha\beta^2\theta}.
\end{aligned}
$$

Similarly,for the agents,

$$
\begin{aligned}
A &= \alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)-\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)} \\
(1-\Phi_{CE}) &= \frac{\alpha\beta(1-\tau_k)}{A} \\
\Phi_{CE} &= 1-\frac{\alpha\beta(1-\tau_k)}{A} \\
n_{CE} &= \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n)+x_{CE}\Phi_{CE}A} \\
x_{CE} &= \frac{(1-\alpha)(1-\beta)}{\alpha\beta^2\theta+(1-\alpha)(1-\beta)}.
\end{aligned}
$$

Only equal factor income taxes under the no externality case, yields the planner's growth rate, except under a very restrictive parametric restriction,

$$
\left(\frac{1-\beta}{\beta}\right)^2 = \theta.
$$

Under this equal factor income taxes are *one* among infinitely many factor income tax combinations that decentralize the planner's growth rate. We can show this as follows.

For growth equalization, we need

$$
n_{CE} = \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n)+x_{CE}\Phi_{CE}A} = n_P.
$$

$$
\begin{aligned}
&\Rightarrow \quad \frac{x_{CE}\Phi_{CE}A}{(1-\tau_n)} = \Phi_P x_P \\
&\Rightarrow \quad \frac{\Phi_{CE}A}{(1-\tau_n)} = \Phi_P \\
&\Rightarrow \quad \frac{A-\alpha\beta(1-\tau_k)}{(1-\tau_n)} = 1-\alpha\beta \\
&\Rightarrow \quad A-\alpha\beta(1-\tau_k) = (1-\alpha\beta)(1-\tau_n)
\end{aligned}
$$

$$\Rightarrow \alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)-\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}-\alpha\beta(1-\tau_k)=(1-\alpha\beta)(1-\tau_n).$$

Hence,

$$(\alpha-\alpha\beta)(1-\tau_k)-(\alpha-\alpha\beta)(1-\tau_n)=\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}$$

which implies

$$(1-\beta)(\tau_n-\tau_k)=\frac{\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}.$$

Clearly, as long as $\frac{(1-\beta)}{\beta}\neq\sqrt{\theta}$, $\tau_n=\tau_k$ always decentralizes planner's growth rates. When $\frac{(1-\beta)}{\beta}=\sqrt{\theta}$, any factor income tax combination decentralizes planner's growth rate. As noted in the text, for $\theta=0.2$, (or $\theta=0.5$, as we have used in our numerical exercise) as in Huffman, the value of $\beta=0.690\,98$ is very small and is not consistent with the literature. (When or $\theta=0.5$, $\beta=0.585\,79$ which is even smaller.). We therefore rule out the possibility of equality.

Appendix D: Agent's Welfare

We know

$$C_t=\Phi_{CE}Y_tA(\tau_k,\tau_n)$$

$$
\begin{aligned}
&\Rightarrow \quad \frac{C_t}{C_{t-1}}=\frac{Y_t}{Y_t}=g_y \\
&\Rightarrow \quad g_{c,CE}=g_{y,CE}.
\end{aligned}
$$

Since g_c is a constant, $C_t = C_0 g_c^t$. On the BGP, the supply of labor is the same across time. We denote welfare by Λ, where,

$$\begin{aligned}
\Lambda &= \sum_{j=0}^{\infty} \beta^t [\log C_t + \log(1 - n_{CE})] \\
\Lambda &= \sum_{j=0}^{\infty} \beta^t \log C_t + \frac{\log(1 - n_{CE})}{1 - \beta}
\end{aligned}$$

$$\Rightarrow \Lambda = \log C_o + \beta \log C_1 + \beta^2 \log C_2 + \beta^3 \log C_3 + \beta^4 \log C_4 + ... + \tfrac{\log(1-n_{CE})}{1-\beta}$$

$$\Rightarrow \Lambda = \frac{\log C_o}{1-\beta} + \frac{\beta^2}{1-\beta} \log g_{c,CE} + \frac{\log(1-n_{CE})}{1-\beta}$$

$$\Rightarrow \Lambda = \tfrac{\log[\Phi_{CE} Y_t A(\tau_k, \tau_n)]}{1-\beta} + \tfrac{\beta^2 \alpha}{(1-\beta)(1-\alpha)} \log g_{c,CE} + \tfrac{\log(1-n_{CE})}{1-\beta}$$

$$\Rightarrow \Lambda = \frac{\log[\Phi_{CE}]}{1-\beta} + \frac{\log[Y_0]}{1-\beta} + \frac{\log[A(\tau_k, \tau_n)]}{1-\beta} + \frac{\beta^2 \alpha}{(1-\beta)(1-\alpha)} \log g_{c,CE} + \frac{\log(1-n_{CE})}{1-\beta}.$$

4 Figures

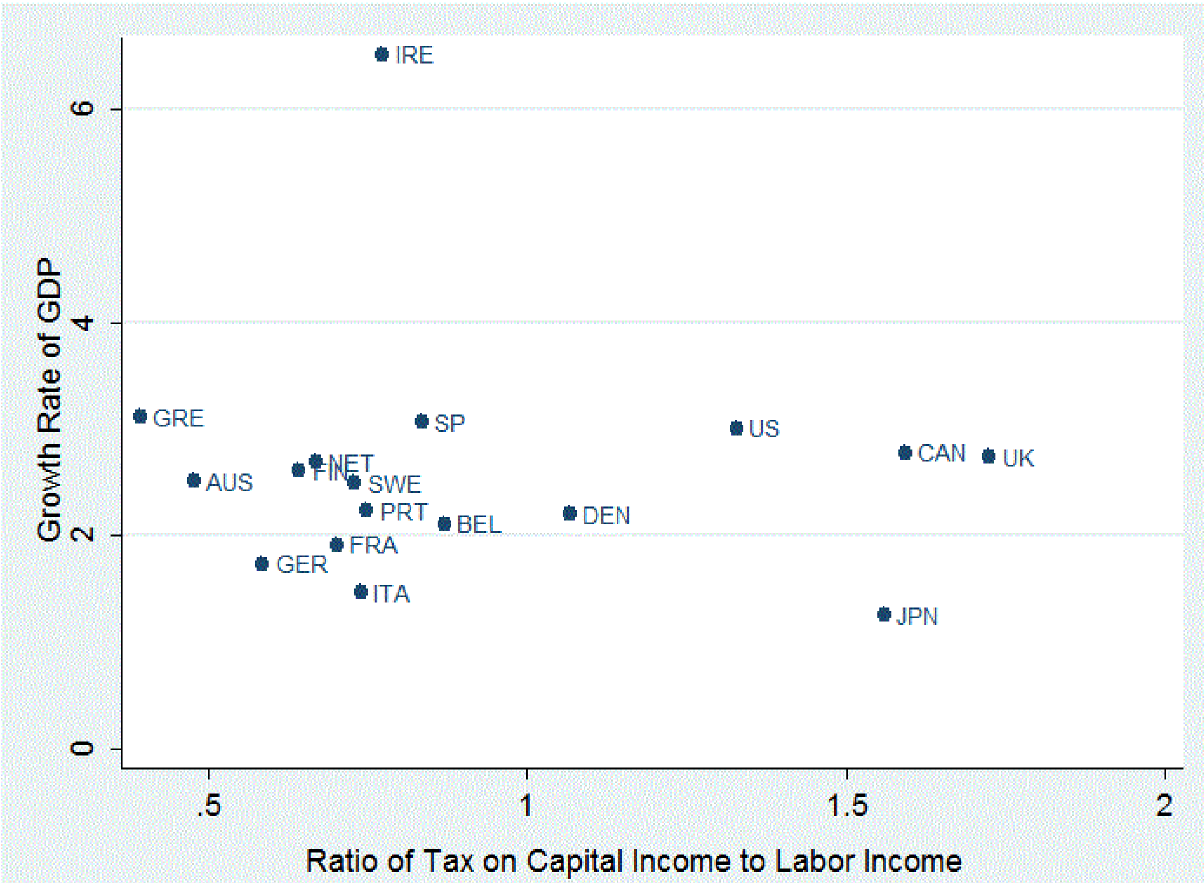

Figure 1: Average growth rates for select OECD economies versus the ratio of tax on capital income to tax on labor income

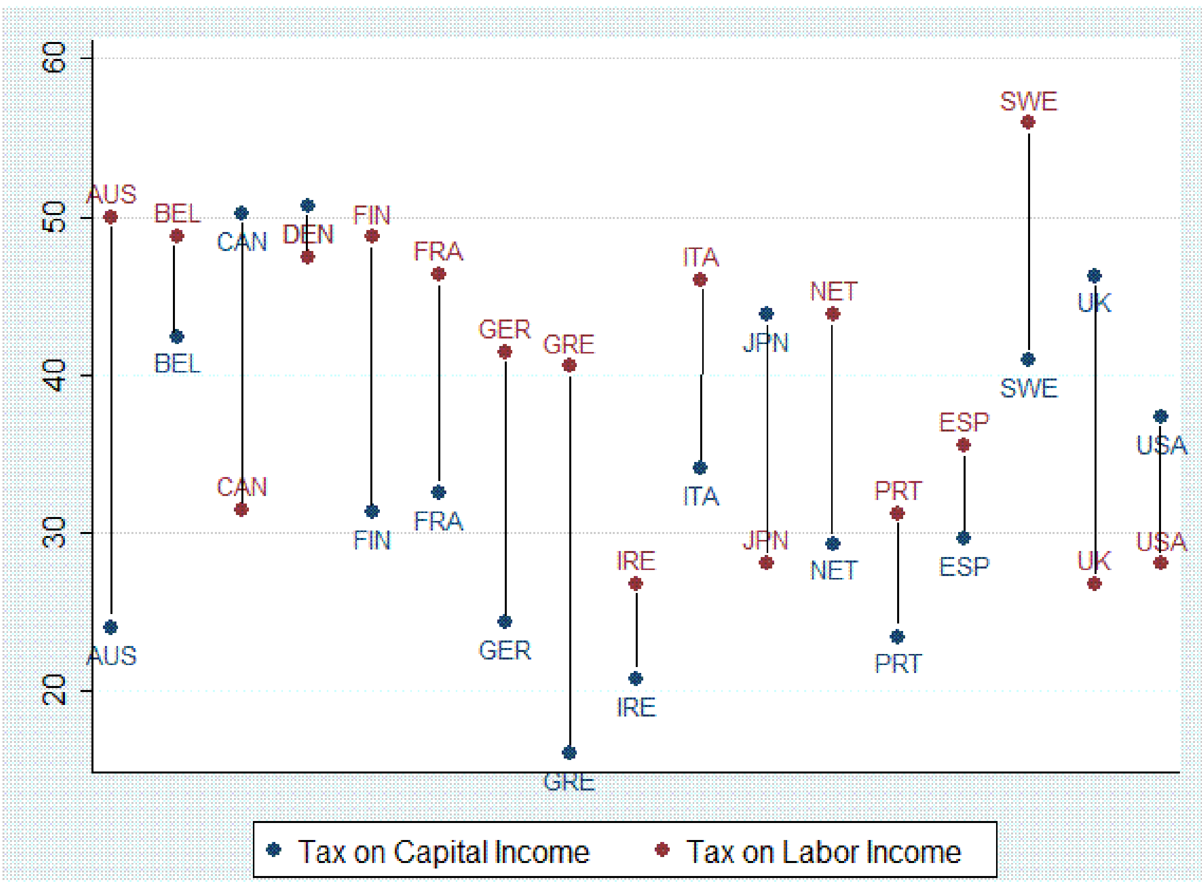

Figure 2: Average factor income tax rates for select OECD economies

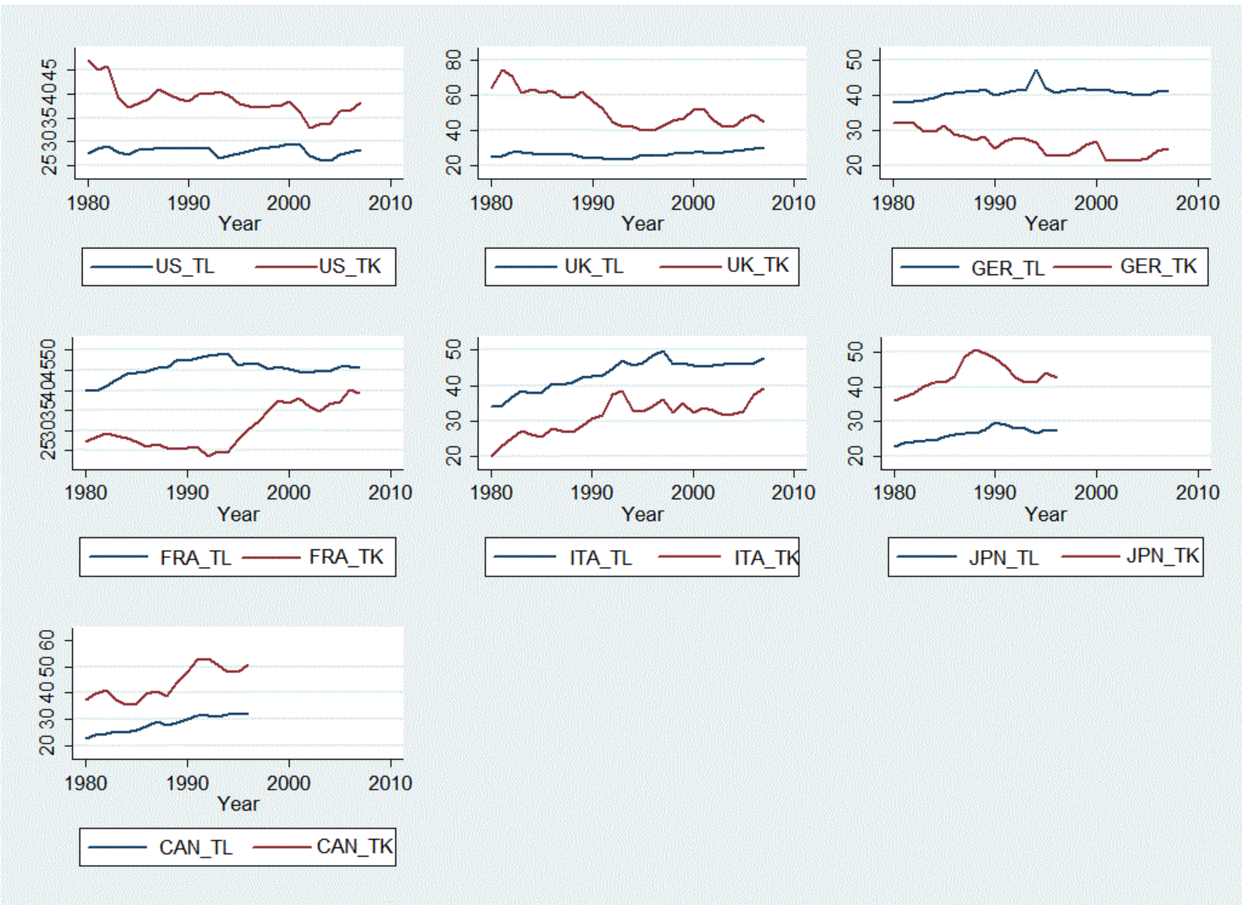

Figure 3: Time trend of factor income taxes for G7 economies

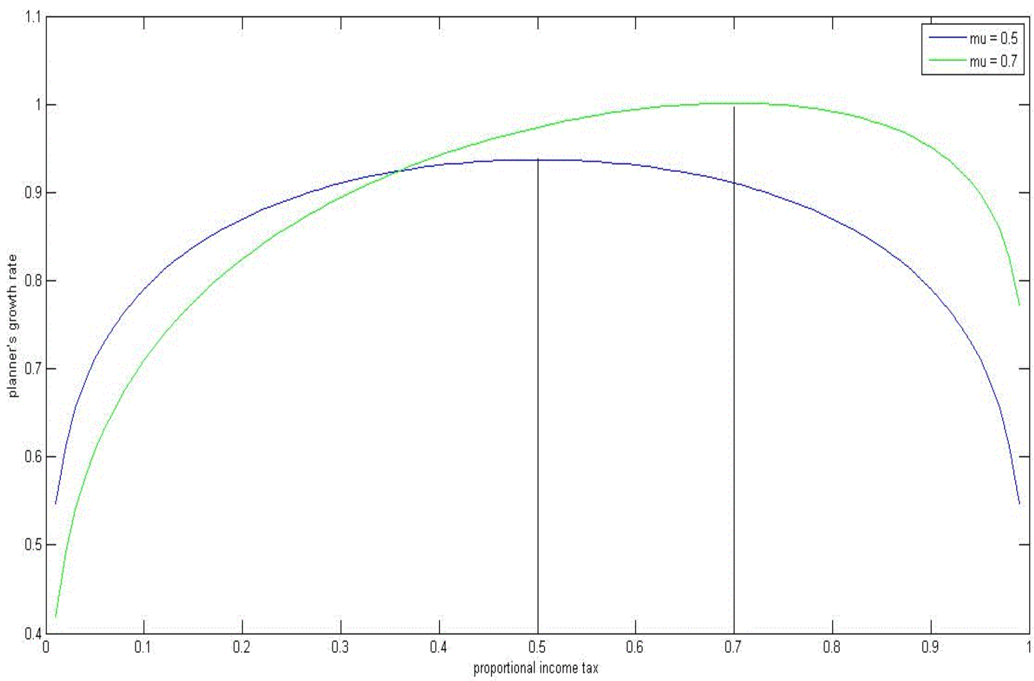

Figure 4: Comparative statics - planner's growth rate

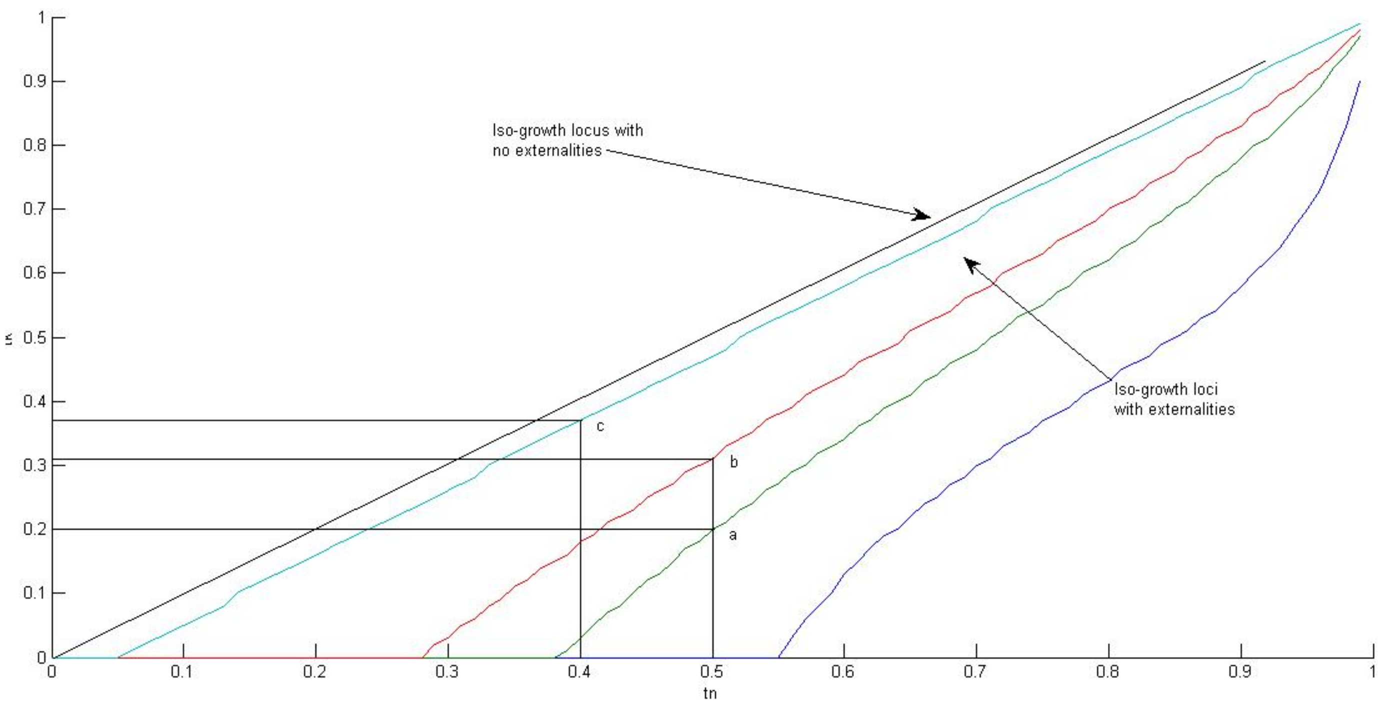

Figure 5: Iso-growth loci for $\xi = 0.1$, $\gamma \to 1$ (with externalities) ; Iso-growth locus for $\xi = 0$ and $\gamma = 1$ (without externalities)

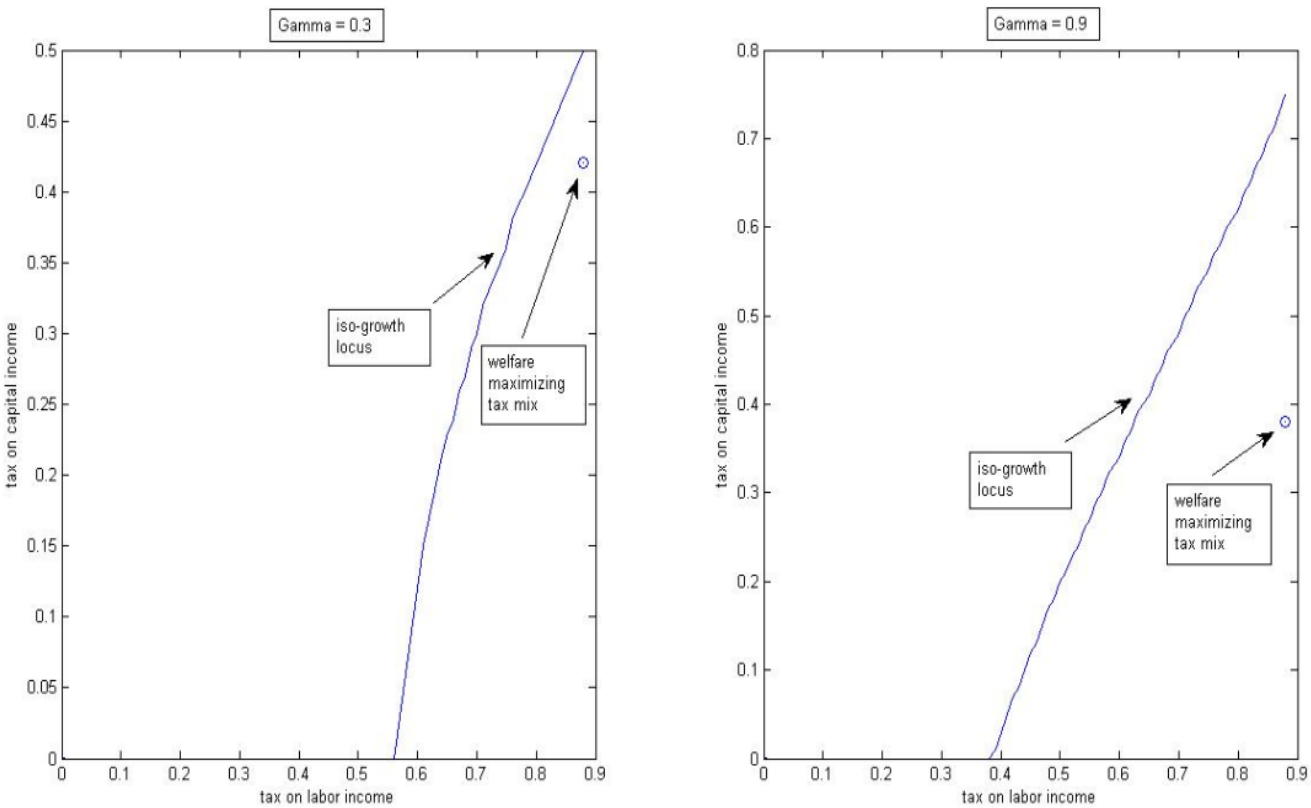

Figure 6: Growth versus welfare: $\xi = 0.1$